Name: ____ Algebra 1

Teacher:_________________________

ALGEBRA 1 SUMMER ASSIGNMENT

Summer 2023

Dear Algebra I Students and Parents:

Welcome to Algebra I! For the 2023-2024 school year, we would like to focus your attention to the prerequisite skills and concepts for Algebra I. In order to be successful for Algebra I, a student must demonstrate a proficiency in:

- The Number System
- Operations with Integers and Fractions
- Solving Equations

The attached review packet is provided for practice. ***Students are expected to have the packet completed when they start school in September.*** As prerequisite skills, most of these topics are <u>not</u> re-taught in the Algebra I course. Teachers will review the answers during the first two or three days the class meets in September. Students are encouraged to seek extra help before or after school from their teacher for any topics requiring more personal, in-depth remediation.

To ensure that all students demonstrate an understanding of the basic Algebra skills to be successful, these topics will be assessed as part of the first Unit Test in September.

It is expected that each student will fully complete the review questions. If you have any questions, please do not hesitate to contact your child's teacher.

In order to determine placement in Algebra 1, several criteria were considered for each student including a placement test score, marking period grades, and teacher recommendations. **Please, note that because this is an advanced class, it is expected that all students will maintain a *C* or higher throughout the year.** If at any point a student's grade drops to a *C* or below, the parents will be contacted by the student's Algebra 1 teacher to discuss strategies to help the student improve their grade. If the student's grade does not improve, the student may be moved to the 8th Grade Introduction to Algebra and Geometry Course.

RESOURCES:

The Number System
https://www.purplemath.com/modules/numtypes.htm

Operations With Integers
https://www.katesmathlessons.com/adding-positive-and-negative-numbers.html

Operations with Fractions
https://www.katesmathlessons.com/adding-and-subtracting-fractions.html
https://www.katesmathlessons.com/multiplying-fractions.html
https://www.katesmathlessons.com/dividing-with-fractions.html

Evaluate Variable Expressions Using Substitution
https://www.katesmathlessons.com/order-of-operations.html

Combining Like Terms
https://www.katesmathlessons.com/adding-and-subtracting-polynomials.html

Solving Equations:

Two Step
https://www.katesmathlessons.com/solving-two-step-equations-p2.html

Variables on Both Sides
https://www.katesmathlessons.com/solving-equations-with-variables-on-both-sides.html

Special Equations – Solutions: $x = 0$, no solutions, infinite solutions (all Real Numbers)
https://www.purplemath.com/modules/solvelin5.htm

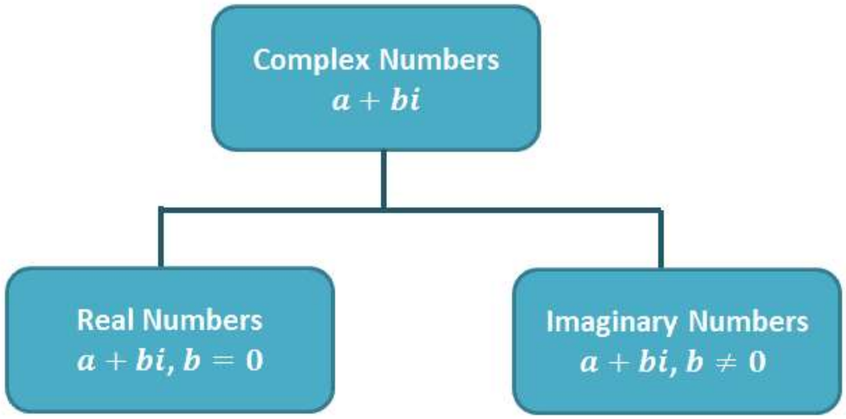
Proportions
https://www.katesmathlessons.com/solving-proportions.html

Rational Equations
https://www.purplemath.com/modules/solvelin3.htm

Complex Number $= a + bi$

a : real number part

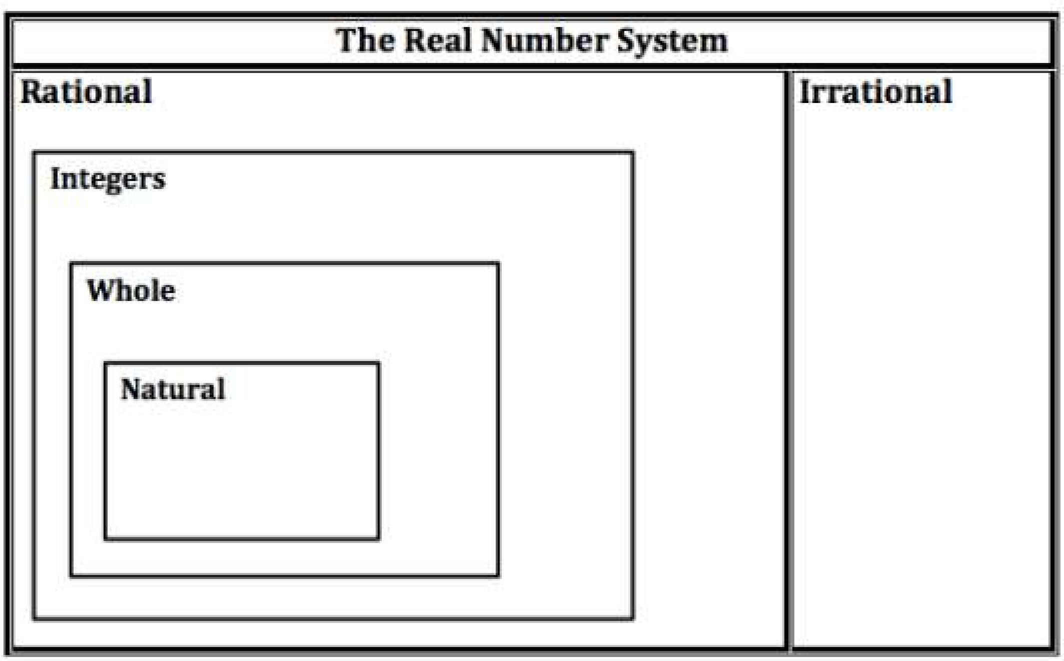
bi : imaginary number part



This chart can help you visualize the group of numbers that belong to the entire complex number system.

- Real numbers such as 4, $\sqrt{5}$, $\frac{4}{2}$, and π all belong to the complex number group as well.
- Imaginary numbers such as $\sqrt{-2}$, $-3 + 2i$, and $5 + 2\sqrt{-6}$ also belong under the complex number system group.

As long as the number can be expressed in the form $a + bi$, it's considered part of the complex number group.



Examples: Mark each box that described the given number.

Number	Natural	Whole	Integer	Rational	Irrational	Imaginary
7	✓	✓	✓	✓		
0		✓	✓	✓		
$2.\overline{37}$				✓		
$\sqrt{2}$					✓	
$\frac{2}{3}$				✓		
-3			✓	✓		
$\sqrt{-2}$						✓

Practice:

Number	Natural	Whole	Integer	Rational	Irrational	Imaginary
1) 25						
2) -14						
3) $\frac{5}{7}$						
4) 0.32						
5) -3.76						
6) $\sqrt{8}$						
7) 23						
8) $\sqrt{100}$						
9) -1.9						
10) 0.47892...						
11) $\sqrt{96}$						
12) $9.\overline{39}$						
13) $\sqrt{-9}$						
14) π						

OPERATIONS WITH INTEGERS

ADDITION:

1) **S**AME SIGNS → FIND THE **S**UM OF THE ABSOLUTE VALUE → KEEP THE SIGN

$2+3=5$ $(-2)+(-3)=(-5)$

2) **D**IFFERENT SIGNS → FIND THE **D**IFFERENCE OF THE ABSOLUTE VALUE → $|>|$ (USE THE SIGN OF THE NUMBER WITH THE GREATER ABSOLUTE VALUE)

$2+(-3)=-1$ $(-2)+3=1$

SUBTRACTION: SUBTRACTION MEANS ADDING THE OPPOSITE:

$2-3=$	$(-2)-3=$	$2-(-3)=$	$(-2)-(-3)=$
$2+(-3)=(-1)$	$(-2)+(-3)=(-5)$	$2+(+3)=5$	$(-2)+(+3)=1$

MULTIPLICATION & DIVISION:

LIKE SIGNS → POSITIVE PRODUCT

$2 \cdot 5 = 10$ $(-2) \cdot (-5) = 10$

UNLIKE SIGNS → NEGATIVE PRODUCT

$(-2) \cdot 5 = (-10)$ $2 \cdot (-5) = (-10)$

OR…COUNT THE NUMBER OF NEGATIVE SIGNS:

EVEN # → POSITIVE PRODUCT $(-2) \cdot 5 \cdot (-1) \cdot (-1) \cdot (-1) = 10$ (4 negative signs)

ODD # → NEGATIVE PRODUCT $(-2) \cdot 5 \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) = (-10)$ (5 negative signs)

FOO – FUNDAMENTAL ORDER OF OPERATIONS

1) GROUPING SYMBOLS: Parentheses ( ) Brackets [ ]

Braces { } Absolute Value Symbol | |

Fraction Bar Radical Sign $\sqrt{\ }$

2) EXPONENTS:

3) MULTIPLICATION & DIVISION: LEFT TO RIGHT (FIRST COME, FIRST SERVED)

4) ADDITION & SUBTRACTION: LEFT TO RIGHT (FIRST COME, FIRST SERVED)

PRACTICE: Simplify each expression. All answers should be written in simplest form.

1) $-2 - 3 - 4$

2) $-5 - 6 + 7$

3) $3 - 4 + 4$

4) $3 - 4 + 5 - 6$

5) $9 - 7 + 5 - (-3)$

6) $-7 - (-7) - 7 - 2 + (-3)$

7) $[-7 \cdot 8 + (-3)] + 4^2$

8) $4(-2)^5 - 21 \div (-7)$

9) $\{(-5 + 1) - [-24 \div (-2)]\}^2$

10) $-12 - 57 \div 3 \div (-1)^9$

OPERATIONS WITH FRACTIONS

Adding Fractions: To add or subtract fractions, you must have a *common denominator*.

$$\frac{1}{3}+\frac{3}{4} \rightarrow \frac{1}{3}\left(\frac{4}{4}\right)+\frac{3}{4}\left(\frac{3}{3}\right) \rightarrow \frac{4}{12}+\frac{9}{12} \rightarrow \frac{13}{12}$$

Multiplying Fractions: To multiply fractions, first cancel where possible. Then, multiply the *numerator* and the *denominator*, then *simplify*.

$$\left(-\frac{2}{7}\right)\left(\frac{1}{4}\right) \rightarrow (-1)\left(\frac{2\cdot 1}{7\cdot 4}\right) \rightarrow \left(-\frac{2}{28}\right) \rightarrow \left(-\frac{1}{14}\right)$$

Dividing Fractions: To divide fractions, multiply the first term by the ***RECIPROCAL*** of the second term.

$$\left(\frac{1}{2}\right)\div\left(\frac{3}{4}\right) \rightarrow \left(\frac{1}{\cancel{2}_{1}}\right)\cdot\left(\frac{\cancel{4}^{2}}{3}\right) \rightarrow \left(\frac{2}{3}\right)$$

PRACTICE: Simplify each expression. All answers should be written in simplest form.

Do not convert improper fractions to mixed numbers!

1) $-\frac{4}{3}+\frac{9}{5}$

2) $-\frac{4}{5}-\frac{5}{8}$

3) $\left(-\frac{4}{3}\right)\left(-\frac{3}{5}\right)$

4) $\left(\frac{10}{7}\right)\left(-\frac{1}{6}\right)$

5) $\left(-\frac{4}{3}\right)\div\left(\frac{3}{5}\right)$

6) $\frac{1}{2}+\left(-\frac{3}{8}\right)$

7) $-\frac{1}{2}-\left(-\frac{3}{8}\right)$

8) $\left(\frac{2}{7}\right)\div\left(\frac{3}{2}\right)$

EVALUATE VARIABLE EXPRESSIONS USING SUBSTITUTION

1) Substitute the given values for each variable.
2) Use PARENTHESES when substituting negative numbers.
3) Simplify using Fundamental Order of Operations.

Evaluate each expression if a = 4, b = -1***, c =*** $\frac{1}{4}$***, and d =*** $\frac{1}{2}$***. Use parentheses when plugging values in!***

1) $2a^2 - 3b + 4$

2) $-3b^3 - 4c + 2d$

3) $\dfrac{ad + d}{d^2}$

4) $\dfrac{a + d}{ac - b}$

5) $\dfrac{abc}{d}$

6) $-b^2 + a^3 - d$

COMBINING LIKE TERMS

Expression: a mathematical phrase that contains operations, numbers and/or variables.

Term: a number, a variable or a product or quotient of numbers or variables that is added or subtracted in an algebraic expression.

- *There are 4 terms in the following expression*: $2x - 4y + 7z + 3$

Variable: a symbol (usually a letter) used to represent a quantity that can change.

Coefficient: a number that is multiplied by a variable.

- *In the term* 2*x,* 2 *is the coefficient. This means* 2 *times the quantity x.*
- *In the term x, the coefficient is understood to be* 1, *even though the number is not written.*

Constant: a term in an algebraic expression that does not change; it does not contain variables.

- *In the expression* $x + 2$, 2 *is a constant.*

Like Term: a term that has the same variable (letter) raised to the same power.

Equivalent: having the same value.

Equation: a mathematical sentence that shows that two expressions are equivalent.

- *The expression* $2x + 7x + 3 - 2$ *can be written as an equivalent expression* $9x + 1$ *after combining like terms.*
- *The expression* $2x - 4y + 7z + 3$ *cannot be simplified because none of the terms are like terms*.

ADDITION AND SUBTRACTION: ONLY LIKE TERMS CAN BE COMBINED.

1) Distribute, if necessary.
2) Combine like terms by adding or subtracting the coefficients of all like terms.

Examples:

a) $3x + 2x = 5x$ b) $3x + 2y$ (CANNOT BE SIMPLIFIED) c) $3x + 10 + 3y + 2x - 2y + 13 = 5x + y + 23$

PRACTICE: Simplify each expression. All answers should be written in simplest form.

1) $5x + 6xy - 7x + 8yx$

2) $3x - 4(2x - 2) - 5$

3) $2(6x + 15) + 3(4x - 10)$

4) $8x - 3x + 9x^2 - 5x + 2x$

5) $2 - (5x - 10) + 7 - x$

6) $(x + 1) - (2x + 3) - (4x - 5)$

SOLVING EQUATIONS

A mathematical sentence with one or more variables is called an **open sentence**. Open sentences are **solved** by finding replacements for the variables that result in true sentences. A sentence that contains an equal sign, =, is called an **equation**.

To solve an equation, find the value or values of a variable that make the equation true. To do this, use inverse operations to **ISOLATE THE VARIABLE**. In other words, get the variable on one side of the equal sign with a coefficient of 1. The Properties of Equality enable us to use inverse operations to isolate the variable and thus solve an equation.

STEPS FOR SOLVING EQUATIONS

1) **Distribute**, if necessary.
2) **Combine Like Terms**, if necessary.
3) **Move the variables** to one side of the equal sign. (Keep coefficient positive, if possible.)
4) **Isolate the variable**. Perform inverse operations to move the constants to the other side of the equation.
5) **Divide by the coefficient** (if the coefficient $\neq 1$)

PRACTICE: Solve for x. ***Express all answers in simplest form. Do not convert improper fraction to mixed numbers!***

1) $\frac{1}{2}x = 10$

2) $4x = -\frac{1}{4}$

3) $10 - x = -10$

4) $-4 - \frac{x}{3} = 1$

5) $8 = -\frac{3}{4}x$

6) $-\frac{2}{3}x + 5 = 13$

7) $\frac{x}{-7} + 2 = 0$

8) $5(4x - 3) = 3$

9) $-5 + 7x = 16$

10) $7 - 3x = -17$

11) $2 + 3(4x - 5) = 6$

12) $6 - 3(4 - 2x) = 30$

EQUATIONS with VARIABLES on BOTH SIDES (*to be reviewed in September)

After simplifying by distributing and combining like terms, use inverse operations to *move the variable terms FIRST* to one side of the equal sign. Then, move the constants to the other side of the equal sign and divide by the coefficient (if it is not equal to 1).

1) $10x + 1 = 15x - 9$

2) $-7x - 8 = 9x - 10$

3) $2(3x - 2) - 5 = 2(2x - 4)$

4) $32 + 2(x - 2) = -3(2x + 4)$

5) $8x - (1 - x) = 7x - 1$

6) $4x + 2(3x - 3) = 4 - 4(2x + 1)$

7) $-(x - 5) - 5 = 3(3x - 10)$

8) $4 + 2(x - 2) = 8 - 3(2x + 4)$

SPECIAL EQUATIONS (*to be reviewed in September)

Sometimes when you solve an equation, the variables are eliminated from the entire equation, and you end up with a statement that can be true or false. If the statement is true, then the solution set = $\Re$. An equation that always produces a true result is called an ***identity***. If the statement is false, then the solutions set = $\varnothing$. An equation that always produces a false result is called a ***contradiction***.

IDENTITY: When solving the equation, the resulting statement is TRUE. → $\Re$

$$\begin{array}{rcl} 6x + 9 &=& 6x + 9 \\ -6x & & -6x \\ \hline 9 &=& 9 \end{array}$$

→ TRUE → x = The Set of Real Numbers → $\Re$

CONTRADICTION: When solving the equation, the resulting statement is FALSE. → $\varnothing$

$$\begin{array}{rcl} 2(3x + 4) &=& 6x - 5 \\ 6x + 8 &=& 6x - 5 \\ -6x & & -6x \\ \hline 8 &=& 5 \end{array}$$

→ FALSE → x = The Null Set → $\varnothing$

PRACTICE: Solve for x. ***Express all answers in simplest form. Do not convert improper fractions to mixed numbers!***

1) $3(x-2) = 3x - 6$

2) $3x + 5 = 5 - 3x$

3) $17 + 8x = 2(4x + 8)$

4) $x + 2 = x$

5) $-x + 3 = -2 - 4x$

6) $3x = 8x - 5x$

PROPORTIONS

A ***proportion*** is an equation that states that two ratios are equivalent. In a proportion, the cross products are equal. If one number in a proportion is unknown, you can find the missing number by finding the cross products and solving the equation. If there is a sum or a difference in a numerator or denominator, use parentheses before you find the cross products (example c).

Examples:

a) $\frac{2}{3} = \frac{10}{15}$ → $2 \cdot 15 = 3 \cdot 10$ → $30 = 30$

b) $\frac{2}{3} = \frac{5}{x}$ → $2x = 3 \cdot 5$ → $2x = 15$ → $x = \frac{7}{2}$

c) $\frac{(2x + 3)}{3} = \frac{5}{2}$ → $2(2x + 3) = 15$ → $4x + 6 = 15$ → $4x = 9$ → $x = \frac{9}{4}$

d) $\frac{2x}{3} = 24$ → $\frac{2x}{3} = \frac{24}{1}$ → $2x = 72$ → $x = 36$

PRACTICE: Solve for x. ***Express all answers in simplest form. Do not convert improper fraction to mixed numbers!***

1) $\frac{2}{x} = \frac{4}{17}$

2) $\frac{2x}{3} = 12$

3) $\frac{4x - 3}{9} = 3$

4) $\frac{x - 2}{2} = \frac{x}{3}$

5) $\frac{x}{7} = \frac{x + 4}{3}$

6) $\frac{2x - 5}{6} = \frac{3x - 1}{3}$

RATIONAL EQUATIONS (*to be reviewed in September)

One strategy for solving an equation with fractions and/or decimals is to multiply the entire equation (ALL TERMS ON BOTH SIDES) by the least common denominator (LCD) to eliminate the fractions. This is sometimes referred to as "sweeping" or "clearing".

Example 1: Solve $\frac{2}{3}x+1=\frac{1}{2}x-2$

$\frac{2}{3}x+1=\frac{1}{2}x-2$ *Given; LCD = 6*

$6\left[\frac{2}{3}x+1=\frac{1}{2}x-2\right]$ *Multiply the ENTIRE equation (EVERY TERM) by the LCD.*

$\left[\frac{6}{1}\cdot\frac{2}{3}x-6\cdot 1=\frac{6}{1}\cdot\frac{1}{2}x-6\cdot 2\right]$

$4x+6=3x-12$ *No more fractions! Solve!*

$x=-18$ *Solution.*

PRACTICE: Solve for x. ***Express all answers in simplest form. Do not convert improper fraction to mixed numbers!***

1) $\frac{1}{5}x-\frac{2}{3}x+\frac{3}{10}x=1$

2) $\frac{5x}{6}-x=\frac{2}{3}$

3) $\frac{2}{5}x-\frac{3}{4}x=\frac{1}{20}$

4) $\frac{1}{2}x-1=\frac{3}{5}x$